

Information Retrieval

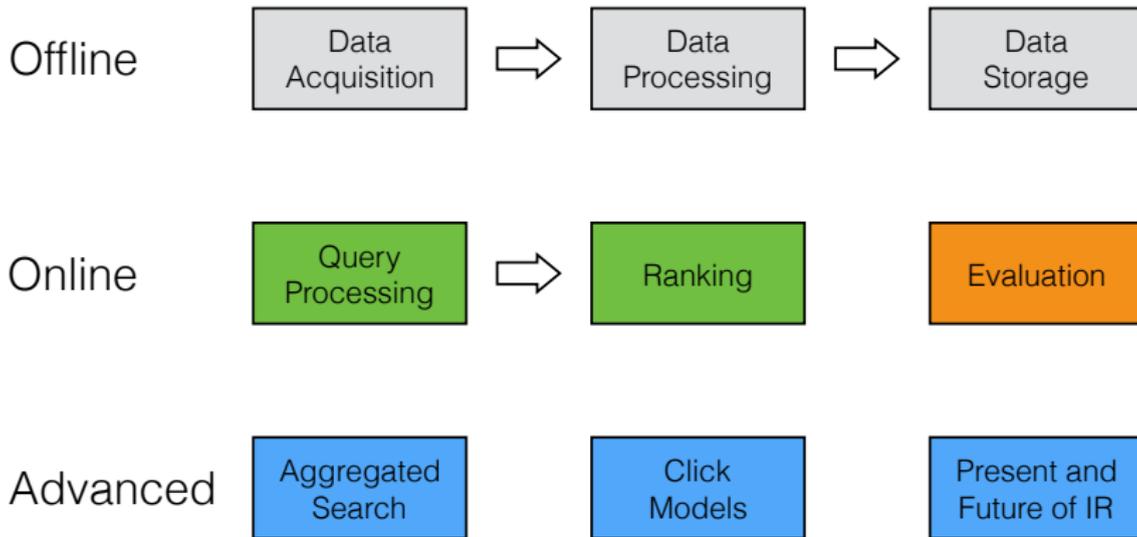
Click Models

Ilya Markov

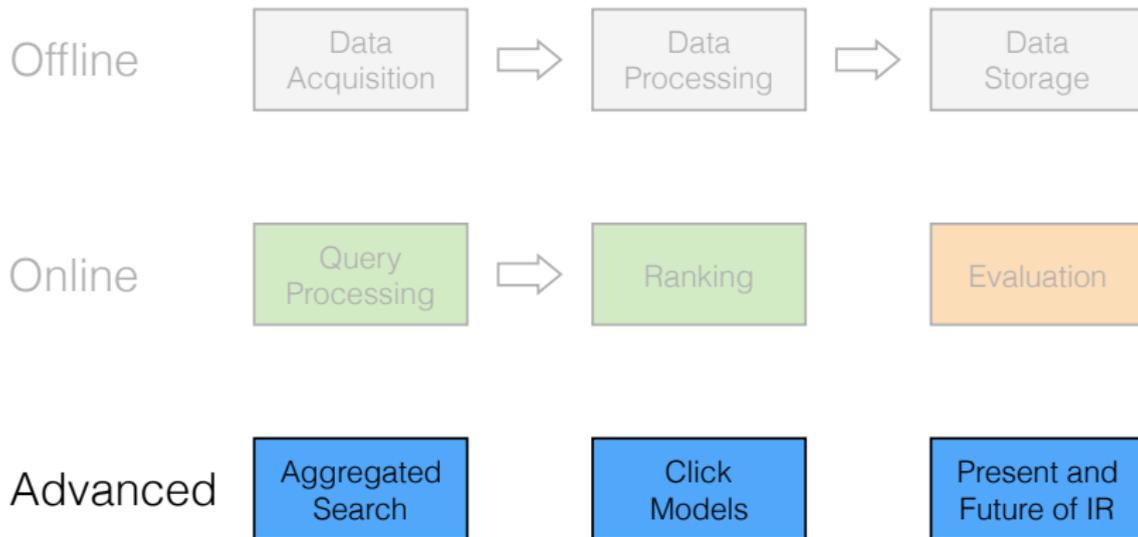
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University of Amsterdam

Course overview



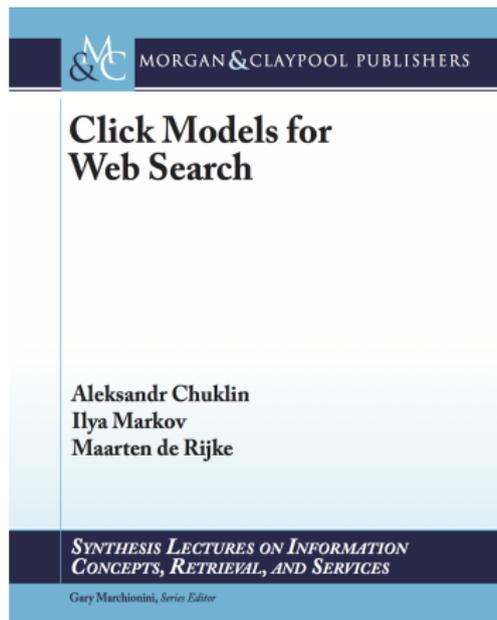
Advanced topics in IR



Outline

- 1 Introduction
- 2 Basic click models
- 3 Applications
- 4 Advanced models
- 5 Current developments
- 6 Future research
- 7 Summary

The book



<http://clickmodels.weebly.com/the-book.html>

Tutorials

- SIGIR 2015, Santiago, Chile
- AINL-ISMW FRUCT 2015, St. Petersburg, Russia
- WSDM 2016, San Francisco, USA
- RuSSIR 2016, Saratov, Russia

`http://clickmodels.weebly.com/tutorials.html`

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Why click models?

Google search results for "wsdm 2016". The search bar contains "wsdm 2016" and the search button is visible. Below the search bar are navigation links: All, News, Images, Videos, Shopping, More, and Search tools. The search results show approximately 61,200 results in 0.44 seconds. The first result is "WSDM 2016 - San Francisco, USA, Feb 22-25, 2016." with a hand cursor clicking on the text. Below this result are links for "WSDM Cup", "Attending", "Call for Papers", "Workshops", and "Sponsors". The second result is "Web Search and Data Mining: The ACM WSDM Conference ..." and the third is "Home - 2016 WSDM Cup Challenge". Below the search results are "Searches related to wsdm 2016" including "sigir 2016", "wsdm 2016 accepted papers", "icde 2016", "wsdm 2014", "wsdm 2017", "what is wsdm", "wsdm 2015", and "wsdm acceptance rate". At the bottom of the page is the "Goooooooooogle" logo with a hand cursor clicking on the letter 'o' at position 6, and a "Next" button.

Why click models?

Scientific modelling is a scientific activity, the aim of which is to make a particular part or feature of the world easier to understand, define, quantify, visualize, or simulate by referencing it to existing and usually commonly accepted knowledge.

Wikipedia, Scientific modelling

Why click models?

Click models make **user clicks** in web search easier to **understand, define, quantify, visualize, or simulate** using (mostly) **probabilistic graphical models**.

Click log

0	0	Q	8	0	7	103	51	92	43	12	73	69	27	105
0	36	Q	174	0	1625	1627	1623	1626	1624	1622	1619	1621	1620	1618
0	50	Q	227	0	2094	2091	2087	2089	2093	2088	2090	2092	2095	2086
0	515	Q	174	0	1625	1627	1623	1626	1624	1622	1619	1621	1620	1618
0	524	Q	1974	0	17562	1627	1626	1623	2091	17559	17563	17558	17561	17560
0	527	C	17562											
0	528	C	1627											
0	529	C	1626											
1	0	Q	9	0	13	70	66	94	50	104	29	21	89	85
1	20	C	104											
1	123	C	21											
1	291	Q	1324	0	11807	11805	11812	11813	11804	11809	11806	11811	11808	11810
1	301	C	11813											
1	8605	C	11808											
1	8737	C	11810											
1	8884	C	11811											
2	0	Q	7	0	77	93	55	86	64	67	76	98	18	54
2	11	C	18											
2	1122	Q	4088	0	35554	35561	35562	35556	35557	35567	35550	35566	35568	35553
2	1127	C	35561											
2	1645	Q	5863	0	36505	36514	36508	36509	50480	36510	36507	50482	50483	50481
2	1646	C	36505											

Yandex Relevance Prediction Challenge
<http://imat-relpred.yandex.ru/en>

Outline

- 1 Introduction
- 2 **Basic click models**
 - Random click model
 - Position-based model
 - Cascade model
 - Click probabilities
 - Evaluation
 - Parameter estimation
- 3 Applications
- 4 Advanced models

Outline

- 2 Basic click models
 - Random click model
 - Position-based model
 - Cascade model
 - Click probabilities
 - Evaluation
 - Parameter estimation

Random click model

The screenshot shows a Yandex search interface. The search bar contains 'san francisco' with 62 million answers. The results are categorized by type: Web, Images, Video, Translate, and More. The first result is 'San Francisco Travel' from sanfrancisco.travel, which is being clicked by a hand cursor. To the right of each result is a label P_{click} .

Web **San Francisco Travel**
[sanfrancisco.travel](#) P_{click}
 San Francisco is home to a bit of everything. Whether you're a first time visitor or a long-time local. This is the place to find out about all things **San Francisco**.

Images

Video

Translate **San Francisco - Wikipedia, the free encyclopedia**
[en.wikipedia.org](#) > [San Francisco](#) P_{click}
San Francisco (/sæn frənˈsɪskooʊ/), officially the City and County of **San Francisco**, is the cultural, commercial, and financial center of Northern California and the only consolidated city-county in California.

More **San Francisco travel guide - Wikitravel**
[wikitravel.org](#) > [en/San_Francisco](#) P_{click}
San Francisco is a major city in California, the centerpiece of the Bay Area, well-known for its liberal community, hilly terrain, Victorian architecture, scenic beauty, summer fog, and great ethnic and cultural diversity.

San Francisco City Guide | Hotels, Restaurants, Nightlife, Real...
[sanfrancisco.com](#) P_{click}
 The job market may seem to navigate these days, but employment and career opportunities can be found in San Francisco's Financial District and Silicon Valley's...

City and County of San Francisco
[sfgov.org](#) P_{click}
 SFGov Visitors Key Services: **SF Travel Resources**. ... Table of links to **San Francisco** districts and supervisors. District. Supervisor.

Random click model

- Terminology
 - C_u – binary random variable denoting a click on document u
- Random click model (RCM)
 - Any document can be clicked with the same (fixed) probability

$$P(C_u = 1) = \text{const} = \rho$$

Random click model





Web

 [San Francisco Travel](#)
[sanfrancisco.travel](#)

San Francisco is home to a bit of everything. Whether you're a first time visitor or a long-time local. This is the place to find out about all things **San Francisco**.

Images

Video

Translate

More

 [San Francisco - Wikipedia, the free encyclopedia](#)
[en.wikipedia.org](#) > [San Francisco](#)

San Francisco (/sænˈfrɑːnsɪkoʊ/), officially the City and County of **San Francisco**, is the cultural, commercial, and financial center of Northern California and the only consolidated city-county in California.

 [San Francisco travel guide - Wikitravel](#)
[wikitravel.org](#) > [en/San_Francisco](#)

San Francisco is a major city in California, the centerpiece of the Bay Area, well-known for its liberal community, hilly terrain, Victorian architecture, scenic beauty, summer fog, and great ethnic and cultural diversity.

 [San Francisco City Guide | Hotels, Restaurants, Nightlife, Real...](#)
[sanfrancisco.com](#)

The job market may seem to navigate these days, but employment and career opportunities can be found in **San Francisco's** Financial District and Silicon Valley's...

 [City and County of San Francisco](#)
[sfgov.org](#)

SFGov Visitors Key Services: **SF** Travel Resources. ... Table of links to **San Francisco** districts and supervisors. District. Supervisor.

$$P(C_{u_1} = 1) = \rho$$

$$P(C_{u_2} = 1) = \rho$$

$$P(C_{u_3} = 1) = \rho$$

$$P(C_{u_4} = 1) = \rho$$

$$P(C_{u_5} = 1) = \rho$$

$$\rho = \frac{\# \text{ clicks}}{\# \text{ shown docs}}$$

CTR models

Random click model (global CTR):

$$P(C_u = 1) = \rho$$

Rank-based CTR:

$$P(C_{u_r} = 1) = \rho_r$$

Query-document CTR:

$$P(C_u = 1) = \rho_{uq}$$

The screenshot shows a Yandex search engine interface. The search bar contains the text "san francisco" and indicates "— 62 million answers". Below the search bar, there are several search results listed under the "Web" category. A mouse cursor is pointing at the first result, "San Francisco Travel", which includes a link to "sanfrancisco.travel" and a snippet: "San Francisco is home to a bit of everything. Whether you're a first time visitor or a long-time local. This is the place to find out about all things San Francisco." Other results include "San Francisco - Wikipedia, the free encyclopedia", "San Francisco travel guide - Wikitravel", "San Francisco City Guide | Hotels, Restaurants, Nightlife, Real...", and "City and County of San Francisco".

Outline

- 2 Basic click models
 - Random click model
 - Position-based model
 - Random click model
 - Click probabilities
 - Evaluation
 - Parameter estimation

Position-based model

Yandex ✕ ↔ Search

Web

 **San Francisco Travel**

sanfrancisco.travel ▾

San Francisco is home to a little bit of everything. Whether you're a first time visitor or a long-time local. This is the place to find out about all things **San Francisco**.

Images

Video

Translate

More

 **San Francisco - Wikipedia, the free encyclopedia**

en.wikipedia.org > [San Francisco](#) ▾

San Francisco (/sænˈfrɑːnˈsikoʊ/), officially the City and County of **San Francisco**, is the cultural, commercial, and financial center of Northern California and the only consolidated city-county in California.

 **San Francisco travel guide - Wikitravel**

wikitravel.org > [en/San Francisco](#) ▾

San Francisco is a major city in California, the centerpiece of the Bay Area, well-known for its liberal community, hilly terrain, Victorian architecture, scenic beauty, summer fog, and great ethnic and cultural diversity.

 **San Francisco City Guide | Hotels, Restaurants, Nightlife, Real...**

sanfrancisco.com ▾

The job market may seem daunting to navigate these days, but employment and career opportunities can be found in **San Francisco's** Financial District and Silicon Valley's...

 **City and County of San Francisco**

sfgov.org ▾

SFGov Visitors Key Services: **SF** Travel Resources. ... Table of links to **San Francisco** districts and supervisors. District. Supervisor.

$$P_{read}(1), P_{click}(u_1q)$$

$$P_{read}(2), P_{click}(u_2q)$$

$$P_{read}(3), P_{click}(u_3q)$$

$$P_{read}(4), P_{click}(u_4q)$$

$$P_{read}(5), P_{click}(u_5q)$$

Position-based model: examination

- Terminology
 - Examination = reading a **snippet**
 - E_r – binary random variable denoting examination of a snippet at rank r
- Position-based model (PBM)
 - Examination depends on rank

$$P(E_r = 1) = \gamma_r$$

Position-based model

Yandex

san francisco — 62 million answers



Search

Web

San Francisco Travel

sanfrancisco.travel ▾

San Francisco is home to a little bit of everything. Whether you're a first time visitor or a long-time local. This is the place to find out about all things **San Francisco**.

Images

Video

Translate

More

San Francisco - Wikipedia, the free encyclopedia

en.wikipedia.org > **San Francisco** ▾

San Francisco (/sænˈfrɑːnsɪskoʊ/), officially the City and County of **San Francisco**, is the cultural, commercial, and financial center of Northern California and the only consolidated city-county in California.

San Francisco travel guide - Wikitravel

wikitravel.org > **en:San Francisco** ▾

San Francisco is a major city in California, the centerpiece of the Bay Area, well-known for its liberal community, hilly terrain, Victorian architecture, scenic beauty, summer fog, and great ethnic and cultural diversity.

San Francisco City Guide | Hotels, Restaurants, Nightlife, Real...

sanfrancisco.com ▾

The job market may seem daunting to navigate these days, but employment and career opportunities can be found in **San Francisco's** Financial District and Silicon Valley's...

City and County of San Francisco

sf.gov.org ▾

SFGov Visitors Key Services: **SF** Travel Resources. ... Table of links to **San** Franisco districts and supervisors. District. Supervisor.

$$\gamma_1, P_{click}(u_1q)$$

$$\gamma_2, P_{click}(u_2q)$$

$$\gamma_3, P_{click}(u_3q)$$

$$\gamma_4, P_{click}(u_4q)$$

$$\gamma_5, P_{click}(u_5q)$$

Position-based model: attractiveness

- Terminology
 - Attractiveness = a user wants to click on a document after examining (reading) its snippet
 - A_{uq} – binary random variable showing whether document u is attractive to a user, given query q
- Position-based model (PBM)
 - Attractiveness depends on a query-document pair

$$P(A_{uq} = 1) = \alpha_{uq}$$

Position-based model

The image shows a Yandex search results page for the query "san francisco". The search bar at the top indicates "62 million answers". Five search results are listed, each with a handwritten label γ_i and a weight $\alpha_{u_i q}$ next to it. Hand-drawn mouse cursor icons point to the first and third search results.

Category	Search Result	Label
Web	San Francisco Travel sanfrancisco.travel San Francisco is home to a little bit of everything. Whether you're a first time visitor or a long-time local. This is the place to find out about all things San Francisco.	$\gamma_1, \alpha_{u_1 q}$
Translate	San Francisco - Wikipedia, the free encyclopedia en.wikipedia.org > San Francisco San Francisco (/sæn frənˈsɪskoo/), officially the City and County of San Francisco, is the cultural, commercial, and financial center of Northern California and the only consolidated city-county in California.	$\gamma_2, \alpha_{u_2 q}$
	San Francisco travel guide - Wikitravel wikitravel.org > en/San_Francisco San Francisco is a major city in California, the centerpiece of the Bay Area, well-known for its liberal community, hilly terrain, Victorian architecture, scenic beauty, summer fog, and great ethnic and cultural diversity.	$\gamma_3, \alpha_{u_3 q}$
	San Francisco City Guide Hotels, Restaurants, Nightlife, Real... sanfrancisco.com The job market may seem difficult to navigate these days, but employment and career opportunities can be found in San Francisco's Financial District and Silicon Valley's...	$\gamma_4, \alpha_{u_4 q}$
	City and County of San Francisco sfgov.org SFGov Visitors Key Services: SF Travel Resources. ... Table of links to San Francisco districts and supervisors. District. Supervisor.	$\gamma_5, \alpha_{u_5 q}$

Position-based model: summary

$$P(E_{r_u} = 1) = \gamma_{r_u}$$

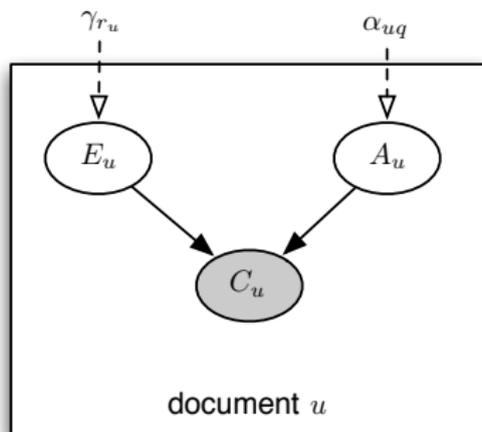
$$P(A_u = 1) = \alpha_{uq}$$

$$P(C_u = 1) = P(E_{r_u} = 1) \cdot P(A_u = 1)$$

Yandex search results for "san francisco" (62 million answers).

- Web**
 - San Francisco Travel**
 - sanfrancisco.travel
 - San Francisco is home to a bit of everything. Whether you're a first time visitor or a long-time local. This is the place to find out about all things San Francisco.
- Images**
- Video**
- Translate**
- More**
 - San Francisco - Wikipedia, the free encyclopedia**
 - en.wikipedia.org
 - San Francisco (often San Fran), officially the City and County of San Francisco, is the cultural, commercial, and financial center of Northern California and the only consolidated city-county in California.
 - San Francisco travel guide - Wikitravel**
 - wikitravel.org
 - San Francisco is a major city in California, the centerpiece of the Bay Area, well-known for its liberal community, hilly terrain, Victorian architecture, scenic beauty, summer fog, and great ethnic and cultural diversity.
 - San Francisco City Guide | Hotels, Restaurants, Nightlife, Real...**
 - sanfrancisco.com
 - The job market may seem to be difficult to navigate these days, but employment and career opportunities can be found in San Francisco's Financial District and Silicon Valley's...
 - City and County of San Francisco**
 - sf.gov
 - SFGov Visitors Key Services: SF Travel Resources. ... Table of links to San Francisco districts and supervisors. District. Supervisor.

Position-based model: probabilistic graphical model



Position-based model: exercises

$$P(E_{r_u} = 1) = \gamma r_u$$

$$P(A_u = 1) = \alpha u_q$$

$$P(C_u = 1) = P(E_{r_u} = 1) \cdot P(A_u = 1)$$

$$E_{r_u} = 0 \Rightarrow C_u = 0$$

$$A_u = 0 \Rightarrow C_u = 0$$

$$E_{r_u} = 1 \Rightarrow (C_u = 1 \iff A_u = 1)$$

$$A_u = 1 \Rightarrow (C_u = 1 \iff E_{r_u} = 1)$$

The screenshot shows a Yandex search engine interface. The search bar contains the text "san francisco" and indicates "62 million answers". Below the search bar, there are several search results categorized by type:

- Web:** "San Francisco Travel" with a URL "sanfrancisco.travel".
- Images:** A snippet of text: "San Francisco is home to a bit of everything. Whether you're a first time visitor or a long-time local. This is the place to find out about all things San Francisco."
- Video:** No results are shown.
- Translate:** "San Francisco - Wikipedia, the free encyclopedia" with a URL "en.wikipedia.org > San Francisco".
- More:** "San Francisco (San Francisco)", "San Francisco is the cultural, commercial, and financial center of Northern California and the only consolidated city-county in California."
- San Francisco travel guide - Wikitravel:** "wikitravel.org > en:San_Francisco".
- San Francisco City Guide | Hotels, Restaurants, Nightlife, Real...:** "sanfrancisco.com".
- City and County of San Francisco:** "sf.gov".

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- 2 Basic click models
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Position-based model

$$P(E_{r_u} = 1) = \gamma_{r_u}$$

$$P(A_u = 1) = \alpha_{uq}$$

$$P(C_u = 1) = P(E_{r_u} = 1) \cdot P(A_u = 1)$$

The screenshot shows a Yandex search interface with the query "san francisco" and approximately 62 million answers. The results are categorized by type:

- Web:**
 - San Francisco Travel** (sanfrancisco.travel): A hand cursor points to this result. The snippet reads: "San Francisco is home to a bit of everything. Whether you're a first time visitor or a long-time local. This is the place to find out about all things San Francisco."
 - San Francisco - Wikipedia, the free encyclopedia** (en.wikipedia.org): A snippet reads: "San Francisco (often *San Fran*), officially the City and County of San Francisco, is the cultural, commercial, and financial center of Northern California and the only consolidated city-county in California."
 - San Francisco travel guide - Wikitravel** (wikitravel.org): A snippet reads: "San Francisco is a major city in California, the centerpiece of the Bay Area, well-known for its liberal community, hilly terrain, Victorian architecture, scenic beauty, summer fog, and great ethnic and cultural diversity."
 - San Francisco City Guide | Hotels, Restaurants, Nightlife, Real...** (sanfrancisco.com): A hand cursor points to this result. The snippet reads: "The job market may seem tough to navigate these days, but employment and career opportunities can be found in San Francisco's Financial District and Silicon Valley's..."
 - City and County of San Francisco** (sf.gov): A snippet reads: "SFGov Visitors Key Services: SF Travel Resources. ... Table of links to San Francisco districts and supervisors. District. Supervisor."

Cascade model

- 1 Start from the first document
- 2 Examine documents one by one
- 3 If click, then stop
- 4 Otherwise, continue

The screenshot shows a Yandex search engine interface. The search bar contains the text "san francisco" and indicates "82 million answers". Below the search bar, there are several search results listed under the "Web" category. A mouse cursor is pointing at the first result, "San Francisco Travel" from "sanfrancisco.travel". The second result is "San Francisco - Wikipedia, the free encyclopedia" from "en.wikipedia.org". The third result is "San Francisco travel guide - Wikitravel" from "wikitravel.org". The fourth result is "San Francisco City Guide | Hotels, Restaurants, Nightlife, Real..." from "sanfrancisco.com". The fifth result is "City and County of San Francisco" from "sf.gov".

Yandex — 82 million answers

Web **San Francisco Travel**
sanfrancisco.travel +
San Francisco is home to a bit of everything. Whether you're a first time visitor or a long-time local. This is the place to find out about all things San Francisco.

Images

Video

Translate

More

San Francisco - Wikipedia, the free encyclopedia
en.wikipedia.org + San Francisco +
San Francisco (Juan Fernández), officially the City and County of San Francisco, is the cultural, commercial, and financial center of Northern California and the only consolidated city-county in California.

San Francisco travel guide - Wikitravel
wikitravel.org + en:San_Francisco +
San Francisco is a major city in California, the centerpiece of the Bay Area, well-known for its liberal community, hilly terrain, Victorian architecture, scenic beauty, summer fog, and great ethnic and cultural diversity.

San Francisco City Guide | Hotels, Restaurants, Nightlife, Real...
sanfrancisco.com +
The job market may seem to navigate these days, but employment and career opportunities can be found in San Francisco's Financial District and Silicon Valley's...

City and County of San Francisco
sf.gov +
SFGov Visitors Key Services: SF Travel Resources... Table of links to San Francisco districts and supervisors. District Supervisor.

Cascade model

$$E_r = 1 \text{ and } A_{u_r} = 1 \Leftrightarrow C_r = 1$$

$$P(A_{u_r} = 1) = \alpha_{u_r} q$$

$$\underbrace{P(E_1 = 1)} = 1$$

start from first

$$\underbrace{P(E_r = 1 \mid E_{r-1} = 0)} = 0$$

examine one by one

$$\underbrace{P(E_r = 1 \mid C_{r-1} = 1)} = 0$$

if click, then stop

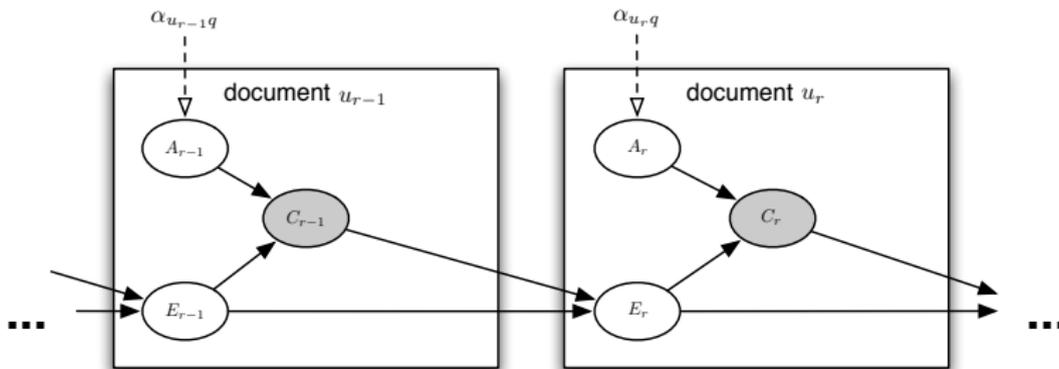
$$\underbrace{P(E_r = 1 \mid E_{r-1} = 1, C_{r-1} = 0)} = 1$$

otherwise, continue

The screenshot shows a Yandex search engine interface. The search bar contains the text "san francisco" and indicates "62 million answers". Below the search bar, there are several search results:

- Web:** "San Francisco Travel" with a snippet: "San Francisco is home to a lot of everything. Whether you're a first time visitor or a long-time local. This is the place to find out about all things San Francisco." A mouse cursor is hovering over this result.
- Images:** (No visible results)
- Video:** (No visible results)
- Translate:** "San Francisco - Wikipedia, the free encyclopedia" with a snippet: "en.wikipedia.org > San Francisco > San Francisco (sæn fræn sɪs'kɔ:), officially the City and County of San Francisco, is the cultural, commercial, and financial center of Northern California and the only consolidated city-county in California." A mouse cursor is hovering over this result.
- More:** "San Francisco travel guide - Wikitravel" with a snippet: "wikitravel.org > en:San_Francisco > San Francisco is a major city in California, the centerpiece of the Bay Area, well-known for its liberal community, hilly terrain, Victorian architecture, scenic beauty, summer fog, and great ethnic and cultural diversity." A mouse cursor is hovering over this result.
- More:** "San Francisco City Guide | Hotels, Restaurants, Nightlife, Real..." with a snippet: "sanfrancisco.com > The job market may seem to navigate these days, but employment and career opportunities can be found in San Francisco's Financial District and Silicon Valley's..." A mouse cursor is hovering over this result.
- More:** "City and County of San Francisco" with a snippet: "sfgov.org > SF Gov Visitors Key Services, SF Travel Resources. ... Table of links to San Francisco districts and supervisors. District Supervisor." A mouse cursor is hovering over this result.

Cascade model: probabilistic graphical model



Basic click models summary

- CTR models
 - + count clicks (simple and fast)
 - do not distinguish examination and attractiveness
- Position-based model (PBM) → **User browsing model**
 - + examination and attractiveness
 - examination of a document at rank r does not depend on examinations and clicks above r
- Cascade model (CM) → **Dynamic Bayesian network**
 - + cascade dependency of examination at r on examinations and clicks above r
 - only one click is allowed

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Click probabilities

- Full probability – probability that a user clicks on a document at rank r

$$P(C_r = 1)$$

- Conditional probability – probability that a user clicks on a document at rank r given previous clicks

$$P(C_r = 1 \mid C_1, \dots, C_{r-1})$$

The screenshot shows a Yandex search engine interface. The search bar contains the text "san francisco" and indicates "62 million answers". Below the search bar, there are several search results categorized by type:

- Web:** "San Francisco Travel" (sanfrancisco.travel) is the top result. A mouse cursor is hovering over the link.
- Images:** "San Francisco is home to a bit of everything. Whether you're a first time visitor or a long-time local. This is the place to find out about all things San Francisco."
- Video:** "San Francisco - Wikipedia, the free encyclopedia" (en.wikipedia.org) is the top result.
- Translate:** "San Francisco (/sæn frænˈsɪskəʊ/), officially the City and County of San Francisco, is the cultural, commercial, and financial center of Northern California and the only consolidated city-county in California."
- More:** "San Francisco travel guide - Wikitravel" (wikitravel.org) is the top result. A mouse cursor is hovering over the link.
- San Francisco City Guide | Hotels, Restaurants, Nightlife, Real...** (sanfrancisco.com) is the top result. A mouse cursor is hovering over the link.
- City and County of San Francisco** (sf.gov) is the top result. A mouse cursor is hovering over the link.

Click probabilities

- Full probability

$$P(C_{r+1} = 1) = \alpha_{u_{r+1}q} \epsilon_r \cdot \left(\begin{array}{l} P(E_{r+1} = 1 \mid E_r = 1, C_r = 1) \cdot P(C_r = 1 \mid E_r = 1) \\ + P(E_{r+1} = 1 \mid E_r = 1, C_r = 0) \cdot P(C_r = 0 \mid E_r = 1) \end{array} \right)$$

- Conditional probability

$$P(C_{r+1} = 1 \mid C_1, \dots, C_r) = \alpha_{u_{r+1}q} \cdot \left(\begin{array}{l} P(E_{r+1} = 1 \mid E_r = 1, C_r = 1) \cdot c_r^{(s)} \\ + P(E_{r+1} = 1 \mid E_r = 1, C_r = 0) \cdot \frac{\epsilon_r(1 - \alpha_{u_rq})}{1 - \alpha_{u_rq}\epsilon_r} \cdot (1 - c_r^{(s)}) \end{array} \right)$$

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Evaluation

Click model's output	Evaluation
Full click probabilities	Perplexity
Conditional click probabilities	Log-likelihood

Perplexity

Perplexity measures how well a click model estimates full click probabilities (i.e., when clicks are not observed).

$$p_r(M) = 2^{-\frac{1}{|S|} \sum_{s \in S} \overbrace{(\log_2 P_M(C_r^{(s)} = c_r^{(s)}))}^{\text{full click probability}})}$$

$$p_r(M) \in [1..2]$$

Likelihood

Likelihood measures how well a click model estimates conditional click probabilities given observed clicks.

$$\begin{aligned}\mathcal{LL}(M) &= \frac{1}{|\mathcal{S}|} \sum_{s \in \mathcal{S}} \log P_M \left(C_1 = c_1^{(s)}, \dots, C_n = c_n^{(s)} \right) \\ &= \frac{1}{|\mathcal{S}|} \sum_{s \in \mathcal{S}} \sum_{r=1}^n \underbrace{\log P_M \left(C_r = c_r^{(s)} \mid \mathbf{C}_{<r} = \mathbf{c}_{<r}^{(s)} \right)}_{\text{conditional click probability}}\end{aligned}$$

$$\mathcal{LL}(M) \in [-\infty..0]$$

Outline

- 2 Basic click models
 - Random click model
 - Position-based model
 - Cascade model
 - Click probabilities
 - Evaluation
 - Parameter estimation

Parameter estimation

- Maximum likelihood estimation
- Expectation-maximization

MLE for random click model

$$P(C_u = 1) = \rho$$

$$\mathcal{L} = \underbrace{\prod_{s \in \mathcal{S}} \prod_{u \in \mathcal{S}} \rho^{c_u^{(s)}} (1 - \rho)^{1 - c_u^{(s)}}}_{\text{likelihood of Bernoulli random variable}}$$

$$\mathcal{L}\mathcal{L} = \sum_{s \in \mathcal{S}} \sum_{u \in \mathcal{S}} \left(c_u^{(s)} \log(\rho) + (1 - c_u^{(s)}) \log(1 - \rho) \right)$$

$$\rho = \frac{\sum_{s \in \mathcal{S}} \sum_{u \in \mathcal{S}} c_u^{(s)}}{\sum_{s \in \mathcal{S}} |s|} = \frac{\# \text{ clicks}}{\# \text{ shown docs}}$$

Expectation maximization

- ① Set parameters to some initial values
- ② Repeat until convergence
 - E-step: derive the expectation of the likelihood function
 - M-step: maximize this expectation

Expectation maximization

$$\begin{aligned}
 Q(\theta_c) &= \sum_{s \in \mathcal{S}} \mathbb{E}_{\mathbf{X} | \mathbf{C}^{(s)}, \Psi} \left[\log P(\mathbf{X}, \mathbf{C}^{(s)} | \Psi) \right] \\
 &= \sum_{s \in \mathcal{S}} \mathbb{E}_{\mathbf{X} | \mathbf{C}^{(s)}, \Psi} \left[\sum_{c_j \in \mathcal{S}} \left(\mathcal{I}(X_{c_j}^{(s)} = 1, \mathcal{P}(X_{c_j}^{(s)}) = \mathbf{p}) \log(\theta_c) + \mathcal{I}(X_{c_j}^{(s)} = 0, \mathcal{P}(X_{c_j}^{(s)}) = \mathbf{p}) \log(1 - \theta_c) \right) + \mathcal{Z} \right] \\
 &= \sum_{s \in \mathcal{S}} \sum_{c_j \in \mathcal{S}} \left(P(X_{c_j}^{(s)} = 1, \mathcal{P}(X_{c_j}^{(s)}) = \mathbf{p} | \mathbf{C}^{(s)}, \Psi) \log(\theta_c) + P(X_{c_j}^{(s)} = 0, \mathcal{P}(X_{c_j}^{(s)}) = \mathbf{p} | \mathbf{C}^{(s)}, \Psi) \log(1 - \theta_c) \right) + \mathcal{Z}
 \end{aligned}$$

$$ESS(x) = \sum_{s \in \mathcal{S}} \sum_{c_j \in \mathcal{S}} P(X_{c_j}^{(s)} = x, \mathcal{P}(X_{c_j}^{(s)}) = \mathbf{p} | \mathbf{C}^{(s)}, \Psi)$$

$$\frac{\partial Q(\theta_c)}{\partial \theta_c} = \sum_{s \in \mathcal{S}} \sum_{c_j \in \mathcal{S}} \left(\frac{P(X_{c_j}^{(s)} = 1, \mathcal{P}(X_{c_j}^{(s)}) = \mathbf{p} | \mathbf{C}^{(s)}, \Psi)}{\theta_c} - \frac{P(X_{c_j}^{(s)} = 0, \mathcal{P}(X_{c_j}^{(s)}) = \mathbf{p} | \mathbf{C}^{(s)}, \Psi)}{1 - \theta_c} \right) = 0$$

$$\begin{aligned}
 \theta_c^{(t+1)} &= \frac{\sum_{s \in \mathcal{S}} \sum_{c_j \in \mathcal{S}} P(X_{c_j}^{(s)} = 1, \mathcal{P}(X_{c_j}^{(s)}) = \mathbf{p} | \mathbf{C}^{(s)}, \Psi)}{\sum_{s \in \mathcal{S}} \sum_{c_j \in \mathcal{S}} \sum_{x=0}^1 P(X_{c_j}^{(s)} = x, \mathcal{P}(X_{c_j}^{(s)}) = \mathbf{p} | \mathbf{C}^{(s)}, \Psi)} \\
 &= \frac{\sum_{s \in \mathcal{S}} \sum_{c_j \in \mathcal{S}} P(X_{c_j}^{(s)} = 1, \mathcal{P}(X_{c_j}^{(s)}) = \mathbf{p} | \mathbf{C}^{(s)}, \Psi)}{\sum_{s \in \mathcal{S}} \sum_{c_j \in \mathcal{S}} P(\mathcal{P}(X_{c_j}^{(s)}) = \mathbf{p} | \mathbf{C}^{(s)}, \Psi)} = \frac{ESS^{(t)}(1)}{ESS^{(t)}(1) + ESS^{(t)}(0)}
 \end{aligned}$$

Click models summary so far

- Basic click models
 - CTR models
 - Position-based model
 - Cascade model
- Click probabilities
 - Full click probabilities
 - Conditional click probabilities
- Evaluation
 - Perplexity
 - Log-likelihood
- Parameter estimation
 - Maximum likelihood estimation
 - Expectation-maximization

What do click models give us?

General:

- Understanding of user behavior

Specific:

- Conditional click probabilities
- Full click probabilities
- Attractiveness and satisfactoriness for query-document pairs

Applications

Click model's output	Application
Understanding of user behavior	User interaction analysis
Conditional click probabilities	User simulation
Full click probabilities	Model-based metrics
Parameter values	Ranking

Outline

- 1 Introduction
- 2 Basic click models
- 3 Applications**
- 4 Advanced models
- 5 Current developments
- 6 Future research
- 7 Summary

User interaction analysis

- Random click model (global CTR): $\rho = 0.122$
- Rank-based CTR:
 $\rho_1 = 0.429, \rho_2 = 0.190, \rho_3 = 0.136, \dots, \rho_{10} = 0.048$
- Position-based model:
 $\gamma_1 = 0.998, \gamma_2 = 0.939, \gamma_3 = 0.759, \dots, \gamma_{10} = 0.260$
- Dynamic Bayesian network model: $\gamma = 0.9997$

Click models are trained on the first 10K sessions of the WSCD 2012 dataset.

Simulating users

Algorithm Simulating user clicks

Input: click model M , query session s

Output: vector of simulated clicks (c_1, \dots, c_n)

- 1: **for** $r \leftarrow 1$ to $|s|$ **do**
 - 2: $P_r \leftarrow P_M(C_r = 1 \mid C_1 = c_1, \dots, C_{r-1} = c_{r-1})$
conditional click probability
 - 3: Generate c_r from *Bernoulli*(P_r)
 - 4: **end for**
-

Model-based metrics

- Utility-based metrics

$$uMetric = \sum_{r=1}^n P(C_r = 1) \cdot U_r$$

- Effort-based metrics

$$eMetric = \sum_{r=1}^n P(S_r = 1) \cdot F_r$$

The screenshot shows a Yandex search engine interface. The search bar contains the text "san francisco" followed by "— 62 million answers". Below the search bar, there are several search results categorized by type:

- Web:**
 - San Francisco Travel** (sanfrancisco.travel) - A hand cursor icon is over the link. Description: "San Francisco is home to a bit of everything. Whether you're a first time visitor or a long-time local. This is the place to find out about all things San Francisco."
- Images:** (No results shown)
- Video:** (No results shown)
- Translate:**
 - San Francisco - Wikipedia, the free encyclopedia** (en.wikipedia.org) - Description: "San Francisco (sən frənˈsɪskoʊ), officially the City and County of San Francisco, is the cultural, commercial, and financial center of Northern California and the only consolidated city-county in California."
- More:**
 - San Francisco travel guide - Wikitravel** (wikitravel.org) - Description: "San Francisco is a major city in California, the centerpiece of the Bay Area, well-known for its liberal community, hilly terrain, Victorian architecture, scenic beauty, summer fog, and great ethnic and cultural diversity."
 - San Francisco City Guide | Hotels, Restaurants, Nightlife, Real...** (sanfrancisco.com) - A hand cursor icon is over the link. Description: "The job market may seem to navigate these days, but employment and career opportunities can be found in San Francisco's Financial District and Silicon Valley's..."
 - City and County of San Francisco** (sf.gov) - Description: "SF Gov Visitors Key Services: SF Travel Resources. ... Table of links to San Francisco districts and supervisors. District, Supervisor."

Expected reciprocal rank

$$\begin{aligned} ERR &= \sum_r \frac{1}{r} \cdot P(S_r = 1) \\ &= \sum_r \frac{1}{r} \cdot R_{u_rq} \cdot \prod_{i=1}^{r-1} (\gamma \cdot (1 - R_{u_iq})) \end{aligned}$$

Features for ranking

Yandex

san francisco — 62 million answers



Search

Web

 **San Francisco Travel**

sanfrancisco.travel ▾

San Francisco is home to a little bit of everything. Whether you're a first time visitor or a long-time local. This is the place to find out about all things **San Francisco**.

$\alpha_{u_1 q}$

Images

Video

Translate

 **San Francisco - Wikipedia, the free encyclopedia**

en.wikipedia.org > **San Francisco** ▾

San Francisco (/sænˈfrɑːnsɪkoʊ/), officially the City and County of **San Francisco**, is the cultural, commercial, and financial center of Northern California and the only consolidated city-county in California.

$\alpha_{u_2 q}$

More

 **San Francisco travel guide - Wikitravel**

wikitravel.org > en/San_Francisco ▾

San Francisco is a major city in California, the centerpiece of the Bay Area, well-known for its liberal community, hilly terrain, Victorian architecture, scenic beauty, summer fog, and great ethnic and cultural diversity.

$\alpha_{u_3 q}$

 **San Francisco City Guide | Hotels, Restaurants, Nightlife, Real...**

sanfrancisco.com ▾

The job market may seem daunting to navigate these days, but employment and career opportunities can be found in **San Francisco's** Financial District and Silicon Valley's...

$\alpha_{u_4 q}$

 **City and County of San Francisco**

sfgov.org ▾

SFGov Visitors Key Services: **SF** Travel Resources. ... Table of links to **San Francisco** districts and supervisors. District. Supervisor.

$\alpha_{u_5 q}$

Outline

- 1 Introduction
- 2 Basic click models
- 3 Applications
- 4 Advanced models**
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Aggregated SERPs







[All](#) [Images](#) [Maps](#) [News](#) [Videos](#) [More](#) [Search tools](#) 

About 679,000,000 results (0,68 seconds)

San Francisco - Wikipedia, the free encyclopedia
https://en.wikipedia.org/wiki/San_Francisco
 San Francisco (/sæn frɪnsɪkoʊ/), officially the City and County of San Francisco, is the cultural, commercial, and financial center of Northern California and ...
 List of people from San - Consolidated city-county - 1906 San Francisco earthquake

San Francisco Travel | Visitor Information
www.sanfrancisco.travel
 Information and features for leisure and business travelers, convention planners, travel trade, and travel media.
[Alcatraz - Things to Do in San Francisco](#) - [Fisherman's Wharf](#) - [Golden Gate Bridge](#)

Images for san francisco Report images



[More images for san francisco](#)

San Francisco City Guide | Hotels, Restaurants, Nightlife ...
www.sanfrancisco.com/
 A complete San Francisco, CA Travel & Tourism Guide specializing in hotels, attractions, restaurants, real estate, nightlife and local business information.

San Francisco Tourism: Best of San Francisco, CA ...
www.tripadvisor.com - United States - California (CA)
 San Francisco Tourism: TripAdvisor has 630023 reviews of San Francisco Hotels, Attractions, and Restaurants making it your best San Francisco resource.

In the news



Twin Peaks shooting victim's family talks about recent arrest
 KGO-TV - 11 hours ago
 San Francisco police confirm they made an arrest in Richmond. However, as of 11 p.m., ...

[Sunny weather brings new temperature record for San Francisco](#)



San Francisco

City in California

San Francisco, in northern California, is a city on the tip of a peninsula surrounded by the Pacific Ocean and San Francisco Bay. It's known for its hilly landscape, year-round fog, iconic Golden Gate Bridge, cable cars and colorful Victorian houses. The Financial District's Transamerica Pyramid is its most distinctive skyscraper. In the bay sits Alcatraz Island, site of the notorious former prison.

Local time: Tuesday 12:38
Weather: 22°C, Wind W at 3 km/h, 51% Humidity
Getting there: 12 h 20 min flight, around CHF 760,00. [View flights](#)

Events

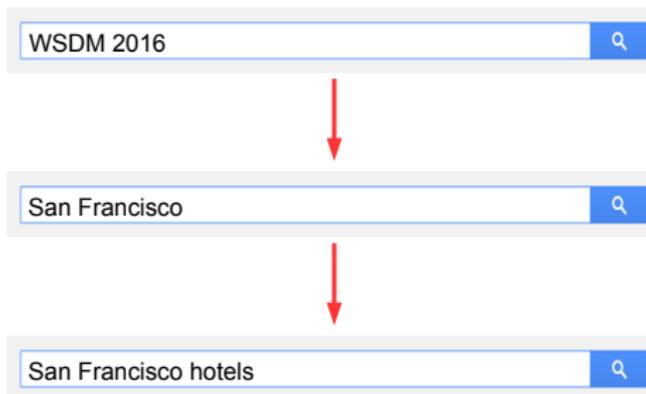
The Wainwright Sisters	Thu 18 Feb
Plastik Funk	Sat 20 Feb
Trevor Noah	Sat 20 Feb
The Masonic	Sat 20 Feb

Points of interest [View 10+ more](#)

 Golden Gate Bridge	 Fisherman's Wharf	 Alcatraz Island	 Golden Gate Park	 Chinatown
---	--	--	--	--

[More about San Francisco](#)

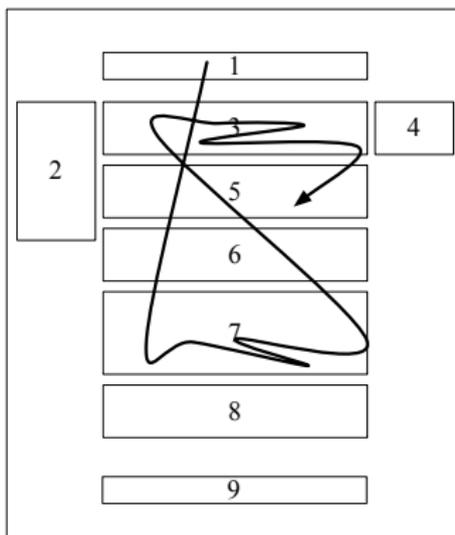
Search tasks



Using features

ID	Feature description
1	Term frequency (TF) of body
2	TF of anchor
3	TF of title
4	TF of URL
5	TF of whole document
6	Inverse document frequency (IDF) of body
7	IDF of anchor
8	IDF of title
9	IDF of URL
10	IDF of whole document
11	TF*IDF of body
12	TF*IDF of anchor
13	TF*IDF of title
14	TF*IDF of URL
15	TF*IDF of whole document
16	Document length (DL) of body
17	DL of anchor
18	DL of title
19	DL of URL
20	DL of whole document
21	BM25 of body
22	BM25 of anchor
23	BM25 of title
24	BM25 of URL
25	BM25 of whole document

Beyond clicks



Picture taken from F. Diaz, R.W. White, G. Buscher, and D. Liebling. Robust models of mouse movement on dynamic web search results pages. In *CIKM*, 2013. ACM Press

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- 7 Summary

Incorporating clicks, attention and satisfaction

Incorporating clicks, attention and satisfaction into a search engine result page evaluation model

Aleksandr Chuklin, Maarten de Rijke

Proceedings of CIKM 2016, Indianapolis, USA

A neural click model for web search

A neural click model for web search

Alexey Borisov, Ilya Markov, Maarten de Rijke, Pavel Serdyukov

Proceedings of WWW 2016, Montreal, Canada

A context-aware time model for web search

A context-aware time model for web search

Alexey Borisov, Ilya Markov, Maarten de Rijke, Pavel Serdyukov

Proceedings of SIGIR 2016, Pisa, Italy
best student paper award

Time between user actions

- **Time between clicks**
- Time to first click
- Time to last click
- Time between queries

Time between clicks

Yandex amsterdam — 191 million answers ✕ ↺ Search

Web  [Amsterdam travel guide - Wikitravel](#)
 wikitravel.com > Amsterdam

Images  Amsterdam, capital of the Netherlands. With more than one million inhabitants in its urban area, it is the country's largest city and its financial, cultural, and creative centre.

Video  Amsterdam gives its name from the city's origin as "Dam" ...

Translate  [Amsterdam - Wikipedia, the free encyclopedia](#)
 en.wikipedia.org > Amsterdam

More  Amsterdam (ˌæmstərˈdæm/; Dutch: [ɑmstərˈdɑm]) is the capital and most populous city in the Kingdom of the Netherlands. Its status as the Dutch capital is mandated by the Constitution of the Netherlands though it is not the seat of the Du...

 [Amsterdam Tourism: Best of Amsterdam, The Netherlands](#)
 tripadvisor.com > Tourism-g188590-Amsterdam_North... >

Amsterdam Tourism: TripAdvisor has 821,053 reviews of **Amsterdam** Hotels, Attractions, and Restaurants making it your best **Amsterdam** resource.

 [Your guide to visit, enjoy, live, work & invest in Amsterdam](#)
 iamsterdam.com > en >

Welcome to **Amsterdam.com**. We would like to ask a few questions about your experience on our website. This will only take a few minutes of your time.

} 30 secs

there is *context bias*

Yandex amsterdam

Web  [Amsterdam travel guide - Wikitravel](#)
 wikitravel.com > Amsterdam

Images  Amsterdam, capital of the Netherlands. With more than one million inhabitants in its urban area, it is the country's largest city and its financial, cultural, and creative centre.

Video  Amsterdam gives its name from the city's origin as "Dam" ...

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 [Amsterdam Tourism: Best of Amsterdam, The Netherlands](#)
 tripadvisor.com > Tourism-g188590-Amsterdam_North... >

Amsterdam Tourism: TripAdvisor has 821,053 reviews of **Amsterdam** Hotels, Attractions, and Restaurants making it your best **Amsterdam** resource.

 [Your guide to visit, enjoy, live, work & invest in Amsterdam](#)
 iamsterdam.com > en >

Welcome to **Amsterdam.com**. We would like to ask a few questions about your experience on our website. This will only take a few minutes of your time.

Modeling time

- Average

$$Time("Amsterdam", "wikipedia.org") = \frac{120 + 60 + 30}{3}$$

- Probability distribution

$$Time("Amsterdam", "wikipedia.org") \sim \text{Gamma}(\mathbf{k}, \boldsymbol{\theta})$$

where $(\mathbf{k}, \boldsymbol{\theta})$ are estimated from 120, 60, 30

context bias is not modeled

Context-aware time modeling

$Time(\text{"Amsterdam"}, \text{"wikipedia.org"}, context_1) \sim Gamma(\mathbf{k}_1, \theta_1)$

$Time(\text{"Amsterdam"}, \text{"wikipedia.org"}, context_2) \sim Gamma(\mathbf{k}_2, \theta_2)$

$Time(\underbrace{\text{"Amsterdam"}, \text{"wikipedia.org"}}_{\text{user action}}, \underbrace{context_3}_{\text{context}}) \sim Gamma(\mathbf{k}_3, \theta_3)$

Context-aware time modeling

$$\begin{aligned} \text{Time}(\text{action}, \text{context}) \sim \text{Gamma}(\quad \\ \mathbf{a}_k(\text{ctx}) \cdot \mathbf{k}(\text{act}) + \mathbf{b}_k(\text{ctx}), \\ \mathbf{a}_\theta(\text{ctx}) \cdot \boldsymbol{\theta}(\text{act}) + \mathbf{b}_\theta(\text{ctx})) \end{aligned}$$

Parameter estimation

$$\begin{aligned} \text{Time}(\text{action}, \text{context}) \sim \text{Gamma}(\quad \\ \mathbf{a}_k(\text{ctx}) \cdot \mathbf{k}(\text{act}) + \mathbf{b}_k(\text{ctx}), \\ \mathbf{a}_\theta(\text{ctx}) \cdot \boldsymbol{\theta}(\text{act}) + \mathbf{b}_\theta(\text{ctx})) \end{aligned}$$

- 1 Fix **context-independent** parameters
- 2 Optimize **context-dependent** parameters using *neural networks*
- 3 Fix **context-dependent** parameters
- 4 Optimize **context-independent** using *gradient descent*
- 5 Repeat until convergence

Parameter estimation

- We do not know the form of **context-dependent** parameters
⇒ neural networks
- We know the form of **context-independent** parameters
(Gamma distribution) ⇒ direct optimization

Context

General	
Is query (Q-action)	(0: no, 1: yes)
Is click (C-action)	(0: no, 1: yes)
$\log(1 + \text{observed time since previous action})$	(0: undefined)
$\log(1 + \text{average time since previous action})$	(0: undefined)
Q-action	
Is new search session	(0: no, 1: yes)
Number of terms in issued query	(0: undefined)
BM25 (issued query, previous query)	(0: undefined)
BM25 (previous query, issued query)	(0: undefined)
C-action	
Is click on the 1 st position	(0: no, 1: yes)
...	...
Is click on the 10 th position	(0: no, 1: yes)

Dataset

3 months of log data from Yandex search engine

Time between actions	Max time	# Observations
Time-to-first-click	1 min	30,747,733
Time-between-clicks	5 min	6,317,834
Time-to-last-click	5 min	30,446,973
Time-from-abandoned-query	1 min	11,523,351

Evaluation tasks

Task1. Predict time between clicks

- Log-likelihood
- Root mean squared error (MSE)

Task2. Rank results based on time between clicks

- $n\text{DCG}@\{1, 3, 5, 10\}$

Task 1. Predicting time

Time model	Distribution	Log-likelihood	RMSE
Basic	exponential	-4.9219	60.73
	gamma	-4.9105	60.76
	Weibull	-4.9077	60.76
Context-aware	exponential	-4.8787	58.93
	gamma	-4.8556	58.98
	Weibull	-4.8504	58.94

Task 2. Ranking results

Time model	Distribution	NDCG			
		@1	@3	@5	@10
Average	—	0.651	0.693	0.728	0.812
Context-aware	exponential	0.668	0.710	0.743	0.820
	gamma	0.675	0.715	0.748	0.822
	Weibull	0.671	0.709	0.745	0.821

Other times

- Time to first click
- Time to last click
- Time between queries

Summary

- Removed **context bias** from time between actions
- Predicted user search interactions better (**Task 1**)
- Used the **context-independent** component for better document ranking (**Task 2**)

Outline

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Future research

- Keep on adding new variables – not a good idea
- Parameter estimation
 - Efficiency
 - Online learning
- Other interactions and environments
 - Interactions beyond clicks
 - Devices beyond desktop computers

Future research

Model's output	Evaluation	Application
Conditional click probs	Log-likelihood	User simulation
Full click probs	Perplexity	Model-based metrics
Parameter values	Ranking evaluation	Ranking

- Why use intermediate evaluation?
 - Evaluate applications, not models
- Why maximize log-likelihood?
 - Optimize models for specific applications

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- 3 Applications
- 4 Advanced models
- 5 Current developments
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- 7 **Summary**

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- 1 Introduction
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Materials

- Aleksandr Chuklin, Ilya Markov, Maarten and de Rijke
Click Models for Web Search
Morgan & Claypool, 2015

Thank you!

